THE UNIVERSITY of EDINBURGH

Edinburgh Research Explorer

Fixed adjustment costs and aggregate fluctuations

Citation for published version:
Elsby, M & Michaels, R 2019, 'Fixed adjustment costs and aggregate fluctuations' Journal of Monetary Economics, vol. 101, pp. 128-147. DOI: 10.1016/j.jmoneco.2018.07.008

Digital Object Identifier (DOI):
10.1016/j.jmoneco.2018.07.008

Published In:
Journal of Monetary Economics

Journal of Monetary Economics 101 (2019) 128–147

Contents lists available at ScienceDirect

Journal of Monetary Economics

journal homepage: www.elsevier.com/locate/jmoneco

Fixed adjustment costs and aggregate fluctuations☆,☆☆,☆☆☆

Michael W.L. Elsby[a,*], Ryan Michaels[b]

[a] School of Economics, University of Edinburgh, 31 Buccleuch Place, Edinburgh EH8 9JT, United Kingdom
[b] Federal Reserve Bank of Philadelphia, USA

ARTICLE INFO

Article history:
Received 18 October 2017
Revised 31 May 2018
Accepted 9 July 2018
Available online 19 July 2018

JEL classification:
E24
E3
J23
J63

Keywords:
Lumpy microeconomic adjustment
Ss model
Cross-sectional dynamics
Aggregate employment dynamics

ABSTRACT

This paper studies the analytics of a canonical model of lumpy microeconomic adjustment. We provide a novel characterization of the implied aggregate dynamics. In general, the distribution of firm outcomes follows a simple and intuitive law of motion that links aggregate outcomes to rates of adjustment. Analytical approximations reveal, however, that the aggregate dynamics are approximately invariant to a relevant range of adjustment costs. This neutrality is an aggregation result that emerges from a symmetry property in the distributional dynamics, independent of market equilibrium considerations. Quantitative illustrations confirm these results for parameterizations used in the employment and price adjustment literatures.

1. Introduction

Inaction in microeconomic adjustment is pervasive. A stylized fact of the empirical dynamics of employment, investment and prices is that they exhibit periods of inaction punctured by bursts of adjustment.[1] A leading explanation of this phenomenon is that firms face a fixed cost of adjusting. In such an environment, firms will choose not to adjust for some time, with periodic discrete adjustments in response to sufficiently large shocks, consistent with the empirical "lumpiness" of microeconomic dynamics.

In this paper, we analyze the aggregate implications of this lumpiness at the microeconomic level. We do so in the context of a canonical model of fixed adjustment costs in the presence of aggregate and idiosyncratic shocks that has been

☆ We are very grateful to Andrew Clausen, Igor Livshits, Ricardo Reis, and an anonymous referee for particularly detailed and helpful comments. We also thank Rudi Bachmann, Giuseppe Bertola, Russell Cooper, William Hawkins, Virgiliu Midrigan, David Ratner, Jonathan Thomas, and anonymous referees, as well as seminar participants at numerous institutions for their comments. All errors are our own. The views expressed here do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia, the staff and members of the Federal Reserve Board, or the Federal Reserve System as a whole.

☆☆ Elsby and Michaels gratefully acknowledge financial support from the UK Economic and Social Research Council (ESRC), Award reference ES/L009633/1.

☆☆☆ Appendices associated with this paper can be found in the Supplementary Material posted online at [WEBSITE].

* Corresponding author.
E-mail address: mike.elsby@ed.ac.uk (M.W.L. Elsby).

[1] See Hamermesh (1989) on employment; Doms and Dunne (1998) on capital; Bertola et al. (2005) on durable goods; and Bils and Klenow (2004) on prices.

https://doi.org/10.1016/j.jmoneco.2018.07.008

used widely in prior literature. For concreteness, we focus on the case of employment adjustment, although we show how the model can be applied equally to price and investment dynamics.

We establish a novel neutrality result: Even in the absence of equilibrium adjustment of market prices, the dynamics of aggregate outcomes implied by standard models are approximately neutral with respect to a plausibly small fixed adjustment cost.

The paper proceeds as follows. In Section 2, we describe the basic ingredients of the model. Firms face shocks to labor productivity that induce changes in their desired level of employment. Firms are subject to both aggregate and idiosyncratic shocks. Aggregate shocks drive macroeconomic expansions and recessions; idiosyncratic shocks drive heterogeneity in employment dynamics across firms. Due to the presence of a fixed adjustment cost, however, firms' employment will not adjust in response to all shocks. Instead, employment evolves according to an *Ss* policy at the microeconomic level, remaining constant for intervals of time with occasional jumps to a new level.

Given this environment, Section 3 takes on the task of aggregating the lumpy microeconomic behavior identified in Section 2 up to the macroeconomic level. These aggregate implications are not obvious. Since individual firms follow highly nonlinear *Ss* labor demand policies, and face heterogeneous idiosyncratic productivities, there is no representative firm interpretation of the model.

We infer the dynamics of aggregate employment by solving for the dynamics of a related object, the cross-sectional distribution of employment across firms. A simple mass-balance approach provides a transparent characterization of the distribution dynamics of employment that holds for a comparatively wide class of processes for shocks and adjustment rules. Perhaps more importantly, our characterization of aggregate dynamics admits a particularly clean economic interpretation. In particular, we show that the evolution of the firm-size distribution can be related simply and intuitively to the probabilities of adjusting to and from each employment level. By impeding these flow probabilities, the adjustment friction distorts the firm-size distribution. These dynamics of the distribution of employment across firms in turn shape the evolution of aggregate employment, since the latter is simply the mean of that distribution.

This characterization of the cross section greatly facilitates our subsequent analysis of the model's dynamics. In Section 4, we develop the main result of the paper—approximate aggregate neutrality. In particular, we use the general results of Section 3 to inform analytical approximations to model outcomes in the presence of a small fixed adjustment cost. This is a compelling neighborhood to study because, as noted since Akerlof and Yellen (1985) and Mankiw (1985), even small adjustment costs will induce substantial inaction in microeconomic adjustment. In this neighborhood, we show that the dynamics of the firm-size distribution approximately coincide with their frictionless counterparts. It follows that the same approximate neutrality extends to the behavior of aggregate employment in general.

We show that this approximate neutrality result can be traced to a symmetry property that emerges in the distributional dynamics of employment as the adjustment friction becomes small. The mass-balance approach of Section 3 makes the intuition for this symmetry particularly transparent. Specifically, the change over time in the density of firms at a given level of employment can be decomposed into an inflow of firms that adjusts to that level, less an outflow of firms that adjust away from that level of employment. The key is that a fixed adjustment cost reduces both of these flows relative to the frictionless case. Fewer firms adjust away from a given employment level. But, in addition, fewer firms find it optimal to adjust to that employment level. For small frictions, these two forces are symmetric, leaving the distribution of employment approximately equal to its frictionless counterpart along its dynamic path.

This neutrality result is reminiscent of Caplin and Spulber (1987) who obtain a similar outcome in a related pricing problem. Although they consider a much simpler environment without idiosyncratic shocks and only one-sided adjustment, our result retains a flavor of theirs. Specifically, Caplin and Spulber demonstrate that a uniform cross-sectional distribution will be invariant in their model due to a form of symmetry—firms induced to adjust from the bottom of the distribution to the top exactly replace firms displaced from the top of the distribution. Thus, one interpretation of our neutrality result is that it generalizes the Caplin and Spulber insight to an environment with idiosyncratic risk and two-sided adjustment. By the same token, this helps to explain why Golosov and Lucas (2007) find small aggregate effects in their quantitative analysis of a related model with these ingredients.

An interesting feature of our approximate aggregate invariance result is that it holds for *any* realization of the aggregate state of the economy, which includes firms' perceptions of the current and future path of the equilibrium wage. That is, it does not rely on equilibrium adjustment of wages.[2] This contrasts with an influential recent literature that has emphasized the role of market price adjustment in muting the aggregate effects of fixed adjustment costs (see, for example, House, 2014; Khan and Thomas, 2008; Veracierto, 2002). Rather, the near-symmetry in the dynamics of the distribution of firm size is a property of *aggregation*, and holds for any configuration of market prices.

In Section 5 of the paper, we illustrate these analytical results in a series of quantitative illustrations. We first parameterize the model using estimates from recent literature on employment adjustment and firm productivity (Bloom, 2009; Cooper et al., 2007; 2015; Foster et al., 2008). Numerical results reveal that this parameterization of the model implies aggregate employment dynamics that are very close to their frictionless analogue even when market wages are fixed, in line with the approximate-neutrality result in Section 4.

[2] In the case of a price setting problem, the aggregate state incorporates firms' anticipations of future aggregate prices. For any set of these anticipations, our neutrality result implies that the aggregate supply curve will approximately coincide with its frictionless counterpart.

There remains a lack of consensus over some of the parameters of the model, however, so we also explore the sensitivity of this baseline result. Alternative parameterizations that match the frequency and average size of employment adjustments in U.S. microdata; vary the persistence of idiosyncratic shocks; and allow for different specifications in which adjustment costs vary stochastically over time, or with firm size, all leave the approximate neutrality result in baseline case essentially unimpaired.

To generate deviations from frictionless dynamics, the model suggests that rates of adjustment must be significantly lower. Consistent with this, we find that the dynamics of aggregate employment can exhibit some persistence relative to its frictionless counterpart in the case where the adjustment cost is larger relative to the variance of innovations to idiosyncratic productivity. This mirrors the emphasis of related research by Alvarez and Lippi (2014) and Alvarez et al. (2016) on the importance of idiosyncratic dispersion to aggregate dynamics. We find, though, that the effects of alternative, plausible parameterizations are modest, yielding only small deviations from frictionless dynamics that vanish after a quarter or two.[3] Taken together, these results suggest that the symmetry result uncovered in Section 4 is quite powerful, in the sense that it is robust to a number of alternative parameterizations.

In the closing sections of the paper, we show how our analytical framework can be used to elucidate cases in which *non*-neutralities emerge. A few recent papers have considered a Poisson-like process for idiosyncratic productivity in which firms draw a new value with some probability each period (Gertler and Leahy, 2008; Midrigan, 2011). This induces an atom in the conditional distribution of idiosyncratic productivity at its lagged value. We show that this discontinuity in turn breaks neutrality in a precise way. Specifically, we show that the symmetry, and hence also the neutrality we emphasize in the early sections of the paper, hold for all firms *except* those prevented from adjusting by the Poisson friction. It follows that aggregate employment evolves approximately according to a pure partial-adjustment process with constant rate of convergence equal to the Poisson parameter, mirroring partial adjustment dynamics. A quantitative illustration confirms the accuracy of this prediction.

We conclude by highlighting promising avenues of future research in the light of our findings. One message is that the role of the magnitude of adjustment frictions relative to idiosyncratic uncertainty in shaping implied aggregate dynamics emphasizes the value of obtaining robust estimates of these parameters. Beyond this, though, the unifying theme of symmetry that underlies the results of this paper provides two further directions to pursue. First, more work that assesses the presence of asymmetries in firms' adjustment policies and their contribution to deviations from frictionless dynamics would be worthwhile. Second, further empirical research into the distributional form of idiosyncratic shocks also will shed an important light on the aggregate consequences of fixed adjustment costs.

2. The firm's problem

We consider a canonical model of fixed employment adjustment costs. Later, we describe how our analysis can be applied to related problems of capital and price adjustment. The microeconomic environment is as follows. Time is discrete. Firms use labor, n, to produce output according to the production function, $y = pxF(n)$, where p represents aggregate productivity, and x represents shocks that are idiosyncratic to an individual firm. We assume the evolution of idiosyncratic shocks is described by the distribution function $G(x'|x)$, with associated density function $g(x'|x)$.[4]

To facilitate the analytical approximations used later in the paper, we make the following assumptions:

A1. *$F(n)$ is analytic, with $F_n(n) > 0$, and $F_{nn}(n) < 0$.*
A2. *$G(x'|x)$ is analytic, and induces the stationary distribution $\mathrm{G}(x') = \int G(x'|x)d\mathrm{G}(x)$.*

The latter assumption is consistent, for example, with conventional parameterizations of idiosyncratic shocks used in the literature, which typically invokes lognormal shocks.

At the beginning of a period, firms observe the realization of their idiosyncratic shocks x, as well as aggregate productivity p. Given this, they then make their employment decision. If the firm chooses to adjust the size of its workforce, it incurs a fixed adjustment cost, denoted C.

For the purposes of the main text, we focus on the case in which there is no exogenous attrition of a firm's workforce, so that during periods of inaction employment remains unchanged. We do this to economize on notation and to convey ideas transparently. The appendices in the Supplementary Material show that all the results we present continue to hold for the case in which a constant fraction δ of the firm's workforce separates each period.

It follows that we can characterize the expected present discounted value of a firm's profits recursively as:

$$\Pi(n_{-1}, x; \Omega) \equiv \max_n \left\{ pxF(n) - wn - C\mathbf{1}^{\Delta} + \beta\mathbb{E}\left[\Pi\left(n, x'; \Omega'\right)|x, \Omega\right]\right\}, \tag{1}$$

where $\mathbf{1}^{\Delta} \equiv \mathbf{1}[n \neq n_{-1}]$ is an indicator that equals one if the firm adjusts and zero otherwise. The wage w is determined in a competitive labor market, and is taken as exogenous from the firm's perspective. The variable Ω summarizes the aggregate

[3] Appendix D in the Supplementary Material also reports results for the case with equilibrium price adjustment. This confirms the message of King and Thomas (2006) and Khan and Thomas (2008) that, where non-neutralities exist for fixed market prices, equilibrium price adjustment pushes the dynamics toward their frictionless path.

[4] We denote lagged values with a subscript, $_{-1}$, and forward values with a prime, $'$.

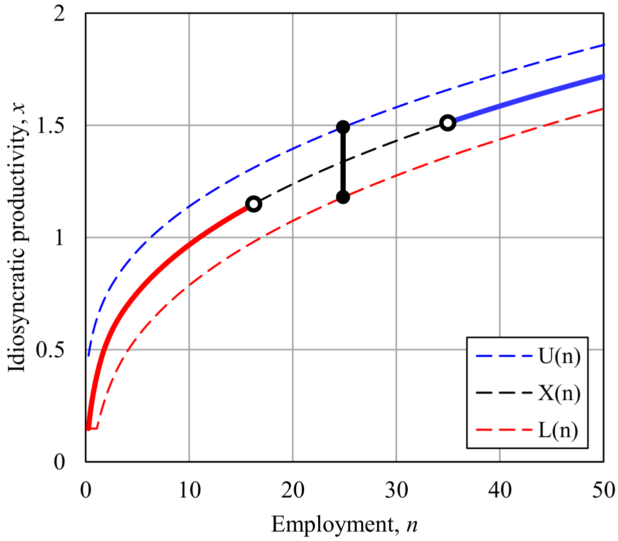

Fig. 1. *Ss* labor demand policy.

state of the economy. The latter includes the wage w, the aggregate shock p, and all variables that are informative with respect to their future evolution, including, for instance, the preceding periods' firm size distributions.

For the analysis that follows, it is helpful to recast the firm's problem in Eq. (1) into two related underlying Bellman equations for the value of adjusting (gross of the adjustment cost), $\Pi^{\Delta}(x;\,\Omega)$, and the value of not adjusting, $\Pi^{0}(n_{-1},x;\Omega)$,

$$\Pi^{\Delta}(x;\,\Omega) \equiv \max_{n}\left\{pxF(n) - wn + \beta\mathbb{E}\left[\Pi\left(n,x';\,\Omega'\right)|x,\,\Omega\right]\right\},\text{ and} \tag{2}$$

$$\Pi^{0}(n_{-1},x;\,\Omega) \equiv pxF(n_{-1}) - wn_{-1} + \beta\mathbb{E}\left[\Pi\left(n_{-1},x';\,\Omega'\right)|x,\,\Omega\right]. \tag{3}$$

Clearly, the value of the firm $\Pi(n_{-1},x;\,\Omega)$ is simply the upper envelope of these two regimes,

$$\Pi(n_{-1},x;\,\Omega) = \max\left\{\Pi^{\Delta}(x;\,\Omega) - C,\,\Pi^{0}(n_{-1},x;\,\Omega)\right\}. \tag{4}$$

It is well-known that it is difficult to characterize in general the optimal policy rule for this problem.[5] In keeping with the literature on fixed adjustment costs, we assume that the optimal labor demand policy takes an *Ss* form. The policy is characterized by three thresholds for the idiosyncratic shock x, $L(n;\,\Omega) < X(n;\,\Omega) < U(n;\,\Omega)$, that determine when to adjust and, if so, by how much. Fig. 1 illustrates such a policy from numerical simulations described later in the paper.

Consider first the question of how to reset employment, conditional on adjusting. Clausen and Strub's (2016) general envelope theorem implies that the firm's problem is differentiable at an optimum, so that the reset policy rule $X(n;\,\Omega)$ satisfies the first-order condition

$$pX(n;\,\Omega)F_n(n) - w + \beta\mathbb{E}\left[\Pi_n\left(n,x',\,\Omega'\right)|x = X(n;\,\Omega),\,\Omega\right] \equiv 0. \tag{5}$$

Thus, $X(n;\,\Omega)$ summarizes labor demand, conditional on adjusting.

Due to the adjustment cost, however, the firm will decide to adjust only if the value of adjusting, net of the adjustment cost, $\Pi^{\Delta}(x;\,\Omega) - C$, exceeds the value of not adjusting, $\Pi^{0}(n_{-1},x;\,\Omega)$. This aspect of the firm's decision rule is characterized by two adjustment thresholds, $L(n_{-1};\,\Omega)$ and $U(n_{-1};\,\Omega)$. For sufficiently bad realizations of the idiosyncratic shock, $x < L(n_{-1};\,\Omega)$, the firm will shed workers; for sufficiently good shocks, $x > U(n_{-1};\,\Omega)$, it will hire workers. For intermediate values of $x \in [L(n_{-1};\,\Omega), U(n_{-1};\,\Omega)]$, the firm will neither hire nor fire, and $n = n_{-1}$. Thus, the adjustment thresholds trace out the locus of points for which the firm is indifferent between adjusting and not adjusting. It follows that the thresholds satisfy the value-matching conditions

$$\begin{aligned}\Pi^{\Delta}(L(n_{-1};\,\Omega);\,\Omega) - C &= \Pi^{0}(n_{-1},L(n_{-1};\,\Omega);\,\Omega),\text{ and}\\ \Pi^{\Delta}(U(n_{-1};\,\Omega);\,\Omega) - C &= \Pi^{0}(n_{-1},U(n_{-1};\,\Omega);\,\Omega).\end{aligned} \tag{6}$$

The following assumption collects the properties of the optimal labor demand policy that we will use throughout:

A3. *Optimal labor demand takes a two-sided* Ss *form in which the thresholds L, X and U are increasing functions of n, and the reset policy X is differentiable in n.*

[5] Exceptions are the continuous-time Brownian case (Harrison et al., 1983), and the case of one-sided adjustment (Roys, 2014; Scarf, 1959).

There are important precedents for this assumption. For example, the influential work of Caballero and Engel (1999) also assumes optimality of a two-sided *Ss* policy. With further assumptions on the scaling of the adjustment cost with productivity, and on the evolution of idiosyncratic shocks, they are able to show that any such policy will have the property that L, X and U are increasing functions of n and analytic.

Our own justification for A3 mirrors that in Gertler and Leahy (2008). As we shall do later in Section 4, Gertler and Leahy study the case of a (plausibly) small C. This implies that the optimal policy in a neighborhood of $C=0$ is indeed *Ss*, with the reset policy X and adjustment thresholds U and L increasing, smooth functions of n.

Firms' optimal policies clearly depend on the aggregate state Ω. For example, positive shocks to p will cause the *Ss* policy in Fig. 1 to shift downward: For any given x, a firm will be less likely to fire, and more likely to hire in an aggregate expansion. However, since many of the ensuing arguments hold for any given aggregate state Ω, to avoid clutter we suppress this notation except where necessary.

3. Aggregation

This section infers the aggregate implications of firms' *Ss* labor demand policies. Aggregation in this context is non-trivial: an individual firm's labor demand depends in a highly nonlinear fashion on its individual lagged employment n_{-1} and the idiosyncratic shock x. The presence of heterogeneity in these state variables implies there is no representative firm interpretation of the model.

To infer aggregate labor demand, we characterize a related object—the cross-sectional distribution of employment across firms. We denote the density of this distribution by $h(n)$, and its associated distribution function by $H(n)$. The aggregation result we develop in this section is an important ingredient to our subsequent analysis in Section 4 of the conditions under which aggregate outcomes are approximately neutral to the adjustment friction, C.

In Proposition 1 we derive the flows in and out of the mass $H(n)$. This in turn implies a law of motion for the density of employment $h(n)$ that has a particularly intuitive form that evokes an "inflow-less-outflow" interpretation. However, since any point along the density function has measure zero, it should be noted that the same is true of these "flows."

This approach can be conveyed most transparently in the special case where x is i.i.d., with distribution function $G(x)$. To begin, we calculate the outflow from the density $h(n)$. The share of firms that adjusts from n is $1-G[U(n)]+G[L(n)]$: the probability that x lies below the lower trigger (which leads the firm to fire) or above the upper trigger (which leads the firm to hire). Therefore, if $h_{-1}(n)$ represents the initial density of firms with employment n, the outflow is

$$(1-G[U(n)]+G[L(n)])\cdot h_{-1}(n). \tag{7}$$

To infer the inflow to $h(n)$, consider the set of firms that draw an idiosyncratic productivity of $x=X(n)$. If the adjustment cost were suspended momentarily, these firms would adjust to n, and the inflow into $h(n)$ would equal $\partial G[X(n)]/\partial n \equiv h^*(n)$. Note that $h^*(n)$ is well-defined by virtue of A2 and A3, which ensure that G and X are differentiable. Following Caballero et al. (1995), we refer to $h^*(n)$ as the density of *mandated* employment.[6] In the presence of a fixed cost, however, Fig. 1 reveals that only firms whose initial employment, n_{-1}, is either relatively low ($n_{-1}<U^{-1}X(n)<n$) or relatively high ($n_{-1}>L^{-1}X(n)>n$) will adjust to n. Thus, the inflow to $h(n)$ is

$$\left(1-H_{-1}\left[L^{-1}X(n)\right]+H_{-1}\left[U^{-1}X(n)\right]\right)\cdot h^*(n), \tag{8}$$

where $H_{-1}(\cdot)$ denotes the distribution function of inherited employment. The change over time in the density at n, $\Delta h(n)$, is then the difference between the inflows (8) and the outflows (7).

Proposition 1 generalizes this approach to the case in which idiosyncratic productivity x follows a first-order Markov process, with distribution function $G(x'|x)$.

Proposition 1 (Aggregation)**.** *The density of employment across firms evolves according to the law of motion*

$$\Delta h(n) = \left(1-\mathcal{H}\left[L^{-1}X(n)|X(n)\right]+\mathcal{H}\left[U^{-1}X(n)|X(n)\right]\right)\cdot h^*(n) - (1-\mathcal{G}[U(n)|n]+\mathcal{G}[L(n)|n])\cdot h_{-1}(n), \tag{9}$$

where $\mathcal{G}(\xi|\nu)\equiv \Pr[x\le\xi|n_{-1}=\nu]$ is the distribution function of idiosyncratic productivity conditional on start-of-period employment; $\mathcal{H}(\nu|\xi)\equiv\Pr[n_{-1}\le\nu|x=\xi]$ is the distribution function of start-of-period employment conditional on idiosyncratic productivity; and $h^*(n)\equiv\partial G[X(n)]/\partial n$ is the density of mandated employment.

Proposition 1 closely resembles the results from the i.i.d. case, except that the probabilities of adjusting to and from n are modified to account for persistence in x. Initial firm size conveys information about past productivity through last period's optimal employment policy. Since productivity is persistent, the probability of events $x\ge U(n)$ or $x\le L(n)$ must then be calculated conditional on initial size, n. It follows that the probability of adjusting *away* from n is $1-\mathcal{G}[U(n)|n]+\mathcal{G}[L(n)|n]$, with $\mathcal{G}$ defined as in Proposition 1. The outflow from n now takes the form in (7), but with G replaced by $\mathcal{G}$. In the same

[6] At this point, the density of employment mandated by the reset policy $X(n)$ in the event that the adjustment cost were suspended *momentarily* may differ from the *frictionless* density of employment that would result if the adjustment cost were suspended *indefinitely*. The reason is that the reset policy can, in principle, depend on the presence of the adjustment cost.

vein, the realization of $x = X(n)$ conveys information about the distribution of lagged employment. Consequently, the probability of adjusting *to* n is evaluated according to the distribution, $\mathcal{H}$, of lagged employment conditional on $x = X(n)$. This yields $1 - \mathcal{H}\left[L^{-1}X(n)|X(n)\right] + \mathcal{H}\left[U^{-1}X(n)|X(n)\right]$. The inflow to n takes the form in (8) but with H_{-1} replaced by $\mathcal{H}$.

To be able to compute the law of motion in Proposition 1 thus requires a characterization of the distribution $\mathcal{G}$, which in turn implies $\mathcal{H}$ by Bayes' rule. This is provided in Lemma 3 in the Supplementary Material, which derives a law of motion for $\mathcal{G}$ and establishes some of its properties.

In summary, Proposition 1 provides a link from the microeconomic friction to the aggregate dynamics. The fixed cost slows the movement of firms away from their initial size n, since a share of them, $\mathcal{G}[U(n)|n] - \mathcal{G}[L(n)|n]$, does not find it profitable to adjust. Likewise, only a fraction of firms that desire to adjust to n relocate there in the face of the fixed cost.

3.1. Aggregate labor demand

With the aid of Proposition 1, it is straightforward to construct aggregate labor demand. Based on the aggregate state Ω, firms derive their optimal labor demand policy functions $L(n;\ \Omega) < X(n;\ \Omega) < U(n;\ \Omega)$. The aggregate implications of firm's choices are expressed through the density of employment $h(n)$, computed as in Proposition 1, for a given history $h_{-1}(n)$. Aggregating over firms thus yields aggregate labor demand for a given aggregate state,

$$N^d(\Omega) = \int nh(n;\Omega)\mathrm{d}n. \tag{10}$$

Proposition 1 delivers a key ingredient to labor market equilibrium. It is only one ingredient, however. Recall that the aggregate state Ω includes aggregate productivity p, the market wage w, and all variables that are informative with respect to their future evolution. Equilibrium requires two additional conditions that bear on Ω, on which Proposition 1 is silent. First, the market wage w adjusts, and is anticipated to adjust, to equate aggregate labor demand in (10) with aggregate labor supply at all points in time. Second, and related, firms' perceptions of the aggregate state Ω must be consistent with equilibrium outcomes. In particular, since Ω includes any information that forecasts future wages, it follows from (10) that firms' perceptions of the current (and expectations of the future) firm-size distribution $h(n)$ are part of the aggregate state. In equilibrium, these perceptions must in turn coincide with the law of motion reported in Proposition 1, evaluated at the equilibrium wage.

Nonetheless, we shall see in Section 4 that Proposition 1 sheds light on the aggregate equilibrium by uncovering properties of aggregate labor demand that hold for *any* Ω. In particular, we establish that aggregate labor demand is approximately invariant to small fixed adjustment costs, in the sense that the aggregate labor demand schedule in Eq. (10) (approximately) coincides with its frictionless counterpart, for *any* set of perceptions about the current (and future evolution) of the aggregate state. It follows that the intersection of aggregate labor demand and supply will yield (approximately) the *frictionless* equilibrium.

3.2. Relation to the literature

We are not the first to consider the analytics of aggregating lumpy microeconomic behavior. For example, a number of papers have considered the implications of *one-sided Ss* policies in which the variable under control—employment in the above model—is adjusted only in one direction. As Cooper et al. (1999) and King and Thomas (2006) show, one-sided adjustment yields much simpler cross-sectional dynamics: Employment (or capital) at each firm decays exogenously, and is intermittently updated to a reset value. However, two-sided adjustment is a perennial feature of employment and price adjustment—firms hire *and* fire workers (Davis and Haltiwanger, 1992); prices are adjusted both up *and* down (Klenow and Malin, 2011). Proposition 1 provides a means to analyze the aggregate effects of adjustment frictions in this empirically-relevant case. We shall see that the presence of two-sided adjustment has important implications for the nature of aggregate dynamics.

In two-sided adjustment problems, progress on aggregation has been made within the class of continuous-time models where idiosyncratic shocks follow a Brownian motion. Indeed, the derivation of the cross-sectional distribution in this context, which applies the Kolomogorov forward equation, resembles the structure of Proposition 1 (see Bertola and Caballero, 1994; Dixit, 1993; Dixit and Pindyck, 1994). Most recently, Alvarez and Lippi (2014) study a price-setting problem with multiple products and show that the dynamics of the average price level—analogous to the mean of $h(n)$ in our context—are mediated by the frequency of adjusting, a result reminiscent of Proposition 1 above. Bertola and Caballero (1990) study aggregate outcomes in a Brownian model in which there are both fixed and kinked costs of adjusting (the latter are omitted in our analysis). Proposition 1 is not restricted to the Brownian class; rather, our results obtain for a general first-order Markov process for idiosyncratic productivity.

For our purposes, Proposition 1 is especially useful because it facilitates analysis of the aggregate dynamics in the next section. The simple link between the dynamics of the cross section and the adjustment probabilities to and from points in the distribution appears to be new to the literature, and provides a mapping from the microeconomic friction to the aggregate dynamics with a clean economic interpretation. We show how to use this result to characterize the model's aggregate implications in a transparent way.

4. Approximate aggregate neutrality

The previous section provided a general characterization of the aggregate dynamics implied by a model of lumpy microeconomic adjustment. In this section, we derive analytical approximations to model outcomes that form the basis of the key result of the paper, namely that the aggregate dynamics characterized in Proposition 1 are approximately neutral with respect to (that is, invariant to) the fixed adjustment cost.

4.1. Some preliminary lemmas

Our analysis in this section begins by describing two intermediate results that inform the neutrality result. These reveal two key properties of the firm's optimal labor demand policy in the neighborhood of a small fixed adjustment cost. The first intermediate result reiterates the insights of Akerlof and Yellen (1985) and Mankiw (1985) to argue that the case of a small fixed cost is particularly instructive, because even small adjustment frictions imply substantial inaction, and hence lumpiness, in microeconomic adjustment. Specifically, the presence of a fixed adjustment cost induces inaction bands that are first order in $C^{1/2}$, and so our approach uses Taylor series expansions of relevant functions in $C^{1/2}$ around the frictionless limit, $C^{1/2}=0$.[7] We denote functions evaluated at $C^{1/2}=0$ by a superscript $\star$; for example, $X^{\star}(\cdot)$ refers to the frictionless reset policy.

Lemma 1. *The adjustment triggers satisfy, for all n,*

$$L(n)=X^{\star}(n)-\gamma(n)C^{1/2}+\Gamma(n)C+O\left(C^{3/2}\right),\text{ and}$$
$$U(n)=X^{\star}(n)+\gamma(n)C^{1/2}+\Gamma(n)C+O\left(C^{3/2}\right),\tag{11}$$

and their inverses satisfy, for all x,

$$L^{-1}(x)=X^{\star^{-1}}(x)+\bar{\gamma}(x)C^{1/2}+\bar{\Gamma}(x)C+O\left(C^{3/2}\right)\text{ , and}$$
$$U^{-1}(x)=X^{\star^{-1}}(x)-\bar{\gamma}(x)C^{1/2}+\bar{\Gamma}(x)C+O\left(C^{3/2}\right),\tag{12}$$

where $\gamma(n)=X^{\star\prime}(n)\bar{\gamma}[X^{\star}(n)]$.

Lemma 1 implies that the adjustment triggers and their inverses that feature prominently in Proposition 1 are approximately symmetric around their corresponding reset rules, with a band of inaction proportional to the square root of the adjustment friction. It follows that even second-order small adjustment costs—that is, $C=\varepsilon^2$—generate first-order inaction bands—for example, $U(n)-L(n)\propto\varepsilon$. The functions $\gamma(n)$ and $\bar{\gamma}(x)$ reflect the curvature in the return to adjusting, and therefore mediate the effect of the adjustment cost on the adjustment triggers. They are linked by the change of variables relation $\gamma(n)=X^{\star\prime}(n)\bar{\gamma}[X^{\star}(n)]$, which maps units of employment to units of productivity.

The second intermediate result we will exploit extends the original insights of Gertler and Leahy (2008) to provide a sharper characterization of the optimal policy. A corollary of their Simplification Theorem for our environment is that the optimal policy approximately coincides with its *myopic* (that is, $\beta=0$) counterpart in the neighborhood of a small fixed adjustment cost. That is, an excellent approximation to optimal dynamic labor demand can be obtained simply by solving for the functions $L(n)$, $X(n)$, and $U(n)$ associated with the corresponding static problem. As stressed by Gertler and Leahy, an important ingredient in this result is the presence of *two-sided adjustment*—that is, that both upward and downward adjustments occur with positive probability in each state. Indeed, Lemma 2 establishes that, if two-sided adjustment obtains, myopia is approximately optimal given any first-order Markov process for x satisfying A2. This generalizes the insight in Gertler and Leahy, whose analysis considered a particular process for idiosyncratic shocks consistent with two-sided adjustment.

Lemma 2. *The expected future value of the firm is independent of current employment n up to third order in* $C^{1/2}$. *The reset policy thus satisfies* $X(n)=X^{\star}(n)+O\left(C^{3/2}\right)$, *for all n.*

The intuition behind the result is straightforward. Note first that current employment affects future profits only in the event that the firm does not adjust in the subsequent period—that is, if $x'\in[L(n),U(n)]$. From Lemma 1, the width of the inaction band is of order $C^{1/2}$. One can show, then, that the probability of inaction is also of order $C^{1/2}$. In addition, by optimality, the return to inaction realized in this event, $\Pi^0\left(n,x'\right)-\left[\Pi^{\Delta}\left(x'\right)-C\right]$, is of order C—it must be bounded from below by zero (otherwise the firm will choose to adjust) and from above by the adjustment cost C (since inaction cannot dominate costless adjustment, $\Pi^0(n,x')\leq\Pi^{\Delta}(x')$). It follows that the effect of n on the future value of the firm, via its role in the expected value of inaction, is of order $C^{3/2}$. The effect of ignoring this term on the firm's profits is negligible, and the firm's problem thus approximates the $\beta=0$ case.

[7] Several authors have explored two-sided adjustment in continuous-time models with Brownian disturbances (see Barro, 1972; Dixit, 1991). In the continuous-time limit, additional smooth pasting conditions pin down the firm's *marginal* value at the adjustment barriers, as the firm faces the prospect of an unboundedly large number of adjustments at these barriers in the presence of Brownian shocks. These conditions in turn imply additional smoothness in the firm's value, and thereby a wider band of inaction that is proportional to $C^{1/4}$ in the continuous-time limit, as opposed to $C^{1/2}$ away from that limit.

A key implication of Lemma 2 for what follows is that the reset policy $X(n)$ approximately coincides with its frictionless counterpart $X^{\star}(n)$, since it approximately satisfies the frictionless first-order condition, $pX^{\star}(n)F_n(n) \equiv w$. An important corollary is that the density of employment *mandated* by the reset policy if the adjustment cost were suspended *momentarily*, $h^{*}(n)$, approximately coincides with the *frictionless* density of employment that would result if the adjustment cost were suspended *indefinitely*, $h^{*}(n) = h^{\star}(n) + O\left(C^{3/2}\right)$.

4.2. The neutrality result

We are now prepared to state the main result of this section, and the key result of the paper, which demonstrates that aggregate dynamics are approximately neutral to the adjustment cost. The differentiability properties summarized in assumptions A1 and A2 facilitate the Taylor series expansions that are used to derive this result. Later, in Section 5, we examine the implications of violations of A2 for a compound-Poisson process for x proposed in recent literature (Gertler and Leahy, 2008; Midrigan, 2011).

Proposition 2 (Neutrality)**.** *The evolution of the density of employment across firms preserves the property*

$$h(n) = h^{\star}(n) + O\left(C^{3/2}\right), \tag{13}$$

for all n and Ω.

Proposition 2 has the following interpretation: If, in some initial period, the density of employment $h_{-1}(n)$ across firms equals its frictionless counterpart $h^{\star}_{-1}(n)$ up to terms that are third order (in $C^{1/2}$), the same will be true of the firm-size density in all subsequent periods, for any sequence of aggregate shocks. For example, imagine that, at an arbitrarily distant date in the past, $C = 0$. Trivially, the firm size density in this period satisfies (13). If a (plausibly small) $C > 0$ is then introduced, and remains in place, Proposition 2 states that (13) will continue to hold. In this sense, the initial condition in (13) is not a strong restriction, insofar as it must hold only at some point in history.

This neutrality result is surprising in a number of respects. First, it is not anticipated by the general representation of aggregation dynamics in Proposition 1. Second, it holds for *any* aggregate state Ω, which includes current and future expectations of market wages. Thus, the neutrality result in Proposition 2 is not the outcome of equilibrium adjustment in wages; it emerges purely from the *aggregation* of microeconomic behavior. Of course, an implication of the latter is that, since neutrality obtains for any Ω, *a fortiori* it also will hold for the equilibrium Ω.

The key to understanding the neutrality result can be traced to a symmetry property in the distributional dynamics of $h(n)$. To see this, it is helpful to rewrite the law of motion for $h(n)$ in Eq. (9) more directly in terms of its constituent flows as

$$\Delta h(n) = \Pr\left(\text{adjust to } n\right)h^{*}(n) - \Pr\left(\text{adjust from } n\right)h_{-1}(n). \tag{14}$$

To see how this sheds light on the source of approximate neutrality, imagine a small fixed adjustment cost is introduced into an otherwise frictionless environment. At any instant of time, the adjustment cost reduces the outflow from any given level of employment n, but also reduces the density of firms which find it optimal to adjust to that level of employment. For small frictions, we show that these two forces are symmetric, leaving the density approximately equal to its frictionless counterpart along the transition path.

It is possible to illustrate this argument more formally if we again assume i.i.d. productivity shocks. Recall that, relative to the frictionless case, the introduction of an adjustment cost reduces the outflow from $h(n)$ by

$$h_{-1}(n)\left(G[U(n)] - G[L(n)]\right). \tag{15}$$

Among firms positioned at n, a share $G[U(n)] - G[L(n)]$ of firms choose not to adjust. Likewise, the inflow to $h(n)$ is reduced at each instant, relative to the frictionless case, by

$$h^{*}(n)\left(H_{-1}\left[L^{-1}X(n)\right] - H_{-1}\left[U^{-1}X(n)\right]\right). \tag{16}$$

Of the density $h^{*}(n)$ of firms for whom n is the mandated level of employment, a share of these firms equal to $H_{-1}\left[L^{-1}X(n)\right] - H_{-1}\left[U^{-1}X(n)\right]$ will choose not to adjust. An approximation to each of the latter expressions around the frictionless optimum reveals that the reductions in both flows converge in the presence of a small adjustment cost. Specifically, noting the form of the adjustment triggers in Lemma 1, Taylor series approximations of $G[U(n)]$ and $G[L(n)]$ in orders of $C^{1/2}$ around $C^{1/2} = 0$ imply that

$$G[U(n)] - G[L(n)] = 2g[X^{\star}(n)]\gamma(n)C^{1/2} + O\left(C^{3/2}\right). \tag{17}$$

Likewise, applying Lemmas 1 and 2 one can show that

$$H_{-1}\left[L^{-1}X(n)\right] - H_{-1}\left[U^{-1}X(n)\right] = 2h_{-1}(n)\bar{\gamma}[X^{\star}(n)]C^{1/2} + O\left(C^{3/2}\right). \tag{18}$$

Recalling the change of variables $\gamma(n) = X^{\star\prime}(n)\bar{\gamma}[X^{\star}(n)]$, and that the mandated density is approximated by its frictionless counterpart $h^{*}(n) = h^{\star}(n) + O\left(C^{3/2}\right)$, where $h^{\star}(n) \equiv X^{\star\prime}(n)g[X^{\star}(n)]$, it follows that the reductions in outflows (15) and inflows (16) converge in the presence of a small adjustment cost, and are given by

$$2h_{-1}(n)h^{\star}(n)\bar{\gamma}[X^{\star}(n)]C^{1/2} + O\left(C^{3/2}\right). \tag{19}$$

It follows that the frictionless density at any given n is preserved along the transition path up to terms of order greater than C.

A key observation is the dual, symmetric roles played by the densities of inherited and frictionless employment levels, $h_{-1}(n)$ and $h^{\star}(n)$, in Eq. (19). Holding constant $h^{\star}(n)$, a large density of inherited employment, $h_{-1}(n)$, implies that a lot of density is "trapped" at n, reducing the *outflow* from that position. But, it also implies that there exists relatively little density at inherited employment levels sufficiently different from n that adjusting to n is optimal, reducing the *inflow*: $h_{-1}(n)$ affects the approximate reduction in outflows and inflows *symmetrically*. Analogously, holding constant $h_{-1}(n)$, a greater density of frictionless employment at n , $h^{\star}(n)$, implies that a smaller density of firms finds it optimal to adjust away from n, reducing the outflow. But, it also will imply that a greater density of firms who would prefer to move to n will be prevented from doing so, reducing the inflow. These two forces offset, and approximate dynamic neutrality obtains.

4.3. The roles of heterogeneity and two-sided adjustment

To develop understanding of Proposition 2, we highlight two further aspects of the neutrality result that sharpen its interpretation. First, Proposition 2 requires that orders of the adjustment cost greater than C be small enough to be considered negligible. Under certain restrictions, there is a more precise metric by which the smallness of C can be evaluated. Consider the family of distributions of idiosyncratic productivity such that $G(x) = \tilde{G}[(x-\mu)/\sigma]$, where μ is a location parameter, and σ a scale parameter that captures dispersion.[8] Then, for example, the reduction in the outflow in Eq. (15) above is given by

$$h_{-1}(n)\left[2\tilde{g}\left(\frac{X^{\star}(n)-\mu}{\sigma}\right)\gamma(n)\left(\frac{C^{1/2}}{\sigma}\right)+O\left(\frac{C^{3/2}}{\sigma^{3}}\right)\right]. \tag{20}$$

Thus, the accuracy of the approximations underlying Proposition 2 hinges on the magnitude of the (square root of the) adjustment cost *relative* to the dispersion of idiosyncratic shocks σ. To see why, recall that the term in brackets is simply the probability of inaction. The approximations obtain if the latter is not very large (though considerable inaction is permitted). The incentive to adjust, in turn, depends on the size of desired adjustments—as governed by the size of changes in productivity—relative to the cost of adjusting. This is captured by $C^{1/2}/\sigma$. Alvarez and Lippi (2014) note the same point using different analytical techniques in a continuous-time Brownian model of price setting. We shall see later that this observation informs our understanding of the quantitative dynamics of the model under alternative calibrations of the adjustment cost C and the dispersion of shocks σ.

The second implication of the neutrality result in Proposition 2 that we wish to highlight is the important role of two-sided adjustment—that is, that there exists a positive probability of both hiring *and* firing workers in each state. To see why this matters, return to the i.i.d. special case, and imagine that the probability of reducing employment $G[L(n)] = 0$ for some employment level n, so that adjustment is one-sided upward. The approximations underlying Lemma 2 and Proposition 2 will fail in this case. The reason is that the inaction rate $G[U(n)] - G[L(n)] = G[U(n)]$ ceases to be (approximately) proportional to $C^{1/2}$, and symmetry is violated.[9]

Any departures from two-sided adjustment in our environment are very limited and, as we shall see, quantitatively unimportant. Here we highlight two examples. First, in the presence of a lump-sum fixed adjustment cost and a lower bound on the distribution of idiosyncratic shocks, it is possible that the lower adjustment trigger $L(n)$ dips below the lower support of x at small employment levels—(very) small firms will adjust only upward. A second, related example is the case in which employment attrits exogenously at rate δ in the absence of adjustment. The Supplementary Material establishes that Lemma 2 and Proposition 2 remain intact under attrition, provided δ is not so large that adjustment becomes one-sided (the firm only hires).[10]

4.4. Applications to capital and price adjustment

Our analysis thus far has been cast in the context of a dynamic labor demand problem. We noted earlier, however, that our results apply equally to canonical models of capital and price adjustment. Here, we briefly explain why. We shall see later that this clear isomorphism aids the comparison of the results noted above with prior literature which spans these related employment, capital and price adjustment problems.

[8] This so-called "location-scale" family of distributions encompasses a variety of commonly-used distributions, including Type-I extreme value, logistic, normal, and exponential distributions, among others.

[9] Specifically, $G[U(n)] - G[L(n)] = G[U(n)] = G[X^{\star}(n)] + g[X^{\star}(n)]\gamma(n)C^{1/2} + O(C)$ in this case, as opposed to $2g[X^{\star}(n)]\gamma(n)C^{1/2} + O\big(C^{3/2}\big)$ in the case of two-sided adjustment.

[10] In their analysis of a menu cost model, Sheshinski and Weiss (1977) show that the optimal reset price is higher when there is drift in the aggregate price level, that is, inflation. The reason drift influences the reset price, and thus the reason symmetry fails to obtain, is that adjustment is one-sided in their model, since there are no idiosyncratic shocks.

4.4.1. Capital adjustment

Reinterpretation of our results for the case of capital adjustment is especially straightforward. The canonical decision problem faced by a firm is given by:

$$\Pi(k_{-1}, x; \Omega) \equiv \max_k \left\{pxF(k) - Rk - C\mathbf{1}^{\Delta} + \beta\mathbb{E}\left[\Pi\left(k, x'; \Omega'\right)|x, \Omega\right]\right\}, \tag{21}$$

where k denotes capital, and R the rental rate on capital.[11] By direct analogy to the labor demand case, the aggregate state Ω will include the rental rate R, aggregate productivity p, and any information pertaining to their future evolution—in particular, perceptions of the current and future distributions of capital. The isomorphism is thus clear: one can pass from (1) to (21) simply by replacing n with k, and w with R. It follows that the equilibrium outcome also will coincide approximately with the frictionless equilibrium.

It is worth re-emphasizing here that the Supplementary Material establishes that approximate neutrality also holds in the presence of depreciation, which is especially applicable to the case of capital adjustment. Depreciation lowers all three policy functions, $L(n)$, $X(n)$ and $U(n)$, in approximately the same way: Firms are more likely to adjust upward, choose higher levels of k conditional on adjusting, and are less likely to adjust downward. This preserves the symmetry of the problem that underlies the neutrality result. Note that the symmetry required for neutrality therefore does not require symmetry of adjustment—neutrality holds in this case even though firms are more likely to adjust upward than downward.

4.4.2. Price adjustment

The problem of price setting under fixed menu costs has a similar structure. Consider a firm facing an isoelastic demand schedule of the form $y = (p/P)^{-\epsilon}Y$, where p is the firm's price; P is the aggregate price level; Y is real aggregate output; and $\epsilon > 1$ is the elasticity of product demand. If the firm operates a linear production function $y = xn$, and faces a market wage w, then one can re-cast the firm's problem as one of choosing the transformed price $q \equiv p^{-\epsilon}$:

$$\Pi(q_{-1}, x; \Omega) \equiv \max_q \left\{Zq^{\alpha} - Z\left(\frac{w}{x}\right)q - C\mathbf{1}^{\Delta} + \beta\mathbb{E}\left[\Pi\left(q, x'; \Omega'\right)|x, \Omega\right]\right\}, \tag{22}$$

where $\alpha \equiv (\epsilon - 1)/\epsilon \in (0, 1)$, and $Z \equiv P^{\epsilon}Y$ is a measure of nominal aggregate demand. Again, the form of (22) has a similar structure to the baseline model of Section 2, but where the aggregate state Ω is now comprised of the market wage w, nominal aggregate demand Z, and perceptions of current and future distributions of prices. Once again, then, the aggregation and neutrality results of Propositions 1 and 2 apply to this pricing problem.

4.5. Relation to the literature

It is instructive to compare our neutrality result in Proposition 2 with related results in the prior literature. Caplin and Spulber (1987) were the first to note the possibility of aggregate neutrality in the presence of lumpy microeconomic adjustment in a related pricing problem. They consider a very simple environment without idiosyncratic shocks and with one-sided *Ss* adjustment. Their ingenious result is that an invariant uniform cross-sectional distribution will be preserved in such an economy, and that aggregate outcomes are unaffected by the adjustment cost: Common shocks move all firms in the same direction in the *Ss* band, and firms induced to adjust at the bottom of the uniform distribution exactly replace those displaced at the top of the distribution.

Our neutrality result in Proposition 2 shares a common theme with Caplin and Spulber's, in the sense that both emerge from a form of symmetry in the model's distributional dynamics. It is interesting that the two models share this theme despite the important difference that we consider an environment with idiosyncratic heterogeneity, and two-sided adjustment.

Golosov and Lucas (2007) add precisely the ingredients of our baseline model to Caplin and Spulber's problem. In their numerical solution of the model, they indeed find very small effects of money on aggregate output. Golosov and Lucas suggest that the robustness of Caplin and Spulber's neutrality result stems from a property of the *Ss* models referred to as the *selection effect*. The idea is that firms that adjust are those that wish to change their price by a lot. Hence, the claim is that, although many firms do not adjust, the aggregate adjustments are large, and neutrality obtains.

The notion of a selection effect from Golosov and Lucas is formalized in the symmetry result underlying Proposition 2. To see this, recall the symmetric effect of $h_{-1}(n)$ on the inflows to and outflows from n. As we noted, if $h_{-1}(n)$ is large, then many firms are "trapped" at n, and outflows from this position are reduced. But, it also means there are many firms near n. These firms are less likely to *select* into n if it is their desired choice, since the small increase in profits does not outweigh the adjustment cost C. This latter, symmetric reduction in the inflows to n is an expression of the selection effect. Hence, our characterization of symmetry in the distributional dynamics formalizes the intuition gleaned from Golosov and Lucas' numerical analysis.

A more recent literature has emphasized the role of equilibrium adjustment in market prices in unwinding the aggregate effects of lumpy adjustment (see House, 2014; Khan and Thomas, 2008; Veracierto, 2002). It is important to note that the neutrality result in Proposition 2 is quite distinct from these channels. Specifically, Proposition 2 suggests that approximate neutrality holds for *any* aggregate state—which includes the wage—that is, regardless of aggregate price movements. What

[11] A standard user cost argument implies that the rental rate R can in turn be related to the price of capital P_k according to $R \equiv P_k - \beta(1-\delta)\mathbb{E}\left[P_k'\right]$.

Table 1
Baseline parameter values.

Parameter	Meaning	Value	Reason
α	Returns to scale	0.64	Cooper et al. (2015)
β	Discount factor	0.99	Quarterly real interest rate = 1%
$C/E(y)$	Adj. cost/Avg. revenue	0.08	Cooper et al. (2015) and Bloom (2009)
ρ_x	Persistence of x	0.70	Cooper et al. (2015) and Foster et al. (2008)
σ_x	Std. dev. of innovation to x	0.35	Cooper et al. (2007, 2015)
ρ_p	Persistence of p	0.95	Autocorrelation of detrended log N
σ_p	Std. dev. of innovation to p	0.015	Std. dev. of detrended log N
δ	Worker attrition rate	0.06	Quarterly quit rate (JOLTS)

is at the heart of Proposition 2 is an *aggregation* result that emerges from the symmetry in the distributional dynamics of $h(n)$.[12]

Finally, recent numerical analyses have found that deviations from frictionless dynamics can be more significant than implied by Proposition 2, if market prices are fixed (Khan and Thomas, 2008; King and Thomas, 2006). Our results suggest that these deviations arise from disruptions of symmetry. In the next section, we show that this can occur when the adjustment cost is large enough relative to idiosyncratic dispersion to violate the approximations underlying Proposition 2. We now turn to these, and related, quantitative issues.

5. Quantitative analysis

Proposition 2 implies that a fixed adjustment cost that induces first-order rates of inaction will induce deviations from frictionless aggregate dynamics that are only third-order. A natural question in the light of this is whether these third-order deviations are also quantitatively small under plausible parameterizations of such models. We address this question in Section 5.1 by parameterizing the model using conventional estimates. We then study the effects of alternative calibrations of the parameters of the model in Section 5.2, and use this to contrast our results with recent quantitative analyses in the related literature. Finally, in Section 5.3 we illustrate analytically how one particular extension of the baseline model can generate aggregate non-neutralities by breaking the symmetry underlying Proposition 2.

5.1. Baseline quantitative analysis

The baseline parameterization we use is summarized in Table 1. The numerical model is cast at a quarterly frequency. We adopt the widespread assumption that the production function takes the Cobb–Douglas form, $F(n) = n^{\alpha}$, with $\alpha < 1$. The returns to scale parameter α is set equal to 0.64 based on estimates reported in Cooper et al. (2007, 2015). This also is similar to the value assumed by King and Thomas (2006). The discount factor β is set to 0.99, which is the conventional choice for a quarterly model.

The magnitude of the adjustment cost is based on estimates reported in Cooper et al. (2015) and Bloom (2009). Cooper et al. (2015) estimate a model similar to the one described above using plant-level data from the Census' Longitudinal Research Database. In one of their better-fitting specifications, they estimate a cost of adjustment equal to 8 percent of quarterly revenue (see row "Disrupt" in their Table 4a). Using annual Compustat data, Bloom (2009) finds nearly the same result, once it is converted to a quarterly frequency (see column "All" in his Table 3). Based on this, we set the adjustment cost parameter C to replicate these estimates. It turns out that this value of C also implies an average frequency of adjustment that is comparable to what is observed in U.S. establishment-level data. In particular, it yields an estimate of the average quarterly probability of adjusting of 56%, as compared to 48.5% in U.S. data.[13]

Idiosyncratic and aggregate shocks are assumed respectively to evolve according to the common assumption of geometric AR(1) processes,

$$\log x' = \mu_x + \rho_x \log x + \varepsilon_x' \text{ , and} \tag{23}$$

$$\log p' = \mu_p + \rho_p \log p + \varepsilon_p', \tag{24}$$

where the innovations are independent normal random variables: $\varepsilon_x' \sim N\left(0, \sigma_x^2\right)$, and $\varepsilon_p' \sim N\left(0, \sigma_p^2\right)$. This baseline parameterization in (23) is again informed by Cooper et al. (2007, 2015), since they recover estimates within related labor demand models. Their estimates of σ_x range from about 0.2 (in their 2007 paper) to 0.5 (in their 2015 paper). We split the difference and set σ_x to be 0.35. However, it has been noted that these papers' estimates of ρ_x, most of which are below 0.5, appear

[12] Of course, this does not preclude that equilibrium price adjustment can weaken the effects of lumpy adjustment on aggregate dynamics in cases where the approximations underlying Proposition 2 do not hold.

[13] This estimate is available from the BLS Business Employment Dynamics program. See http://www.bls.gov/bdm/bdsoc.htm. We take the average over the full sample, 1992q3 to 2013q2.

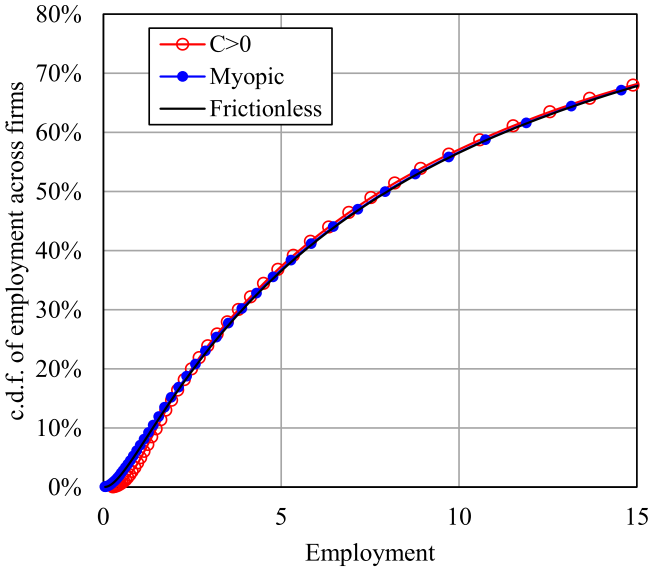

Fig. 2. Steady-state $H(n)$.

rather low relative to other estimates in the literature; Cooper and Haltiwanger (2006) and Foster et al. (2008) each recover estimates of ρ_x near 0.95. Again, we split the difference and set $\rho_x = 0.7$, which is close to the midpoint of this wider range of estimates, and also close to that estimated in Abraham and White (2006) using U.S. manufacturing data. This baseline parameterization is comparable to that used in Bachmann's (2013) analysis of non-convex adjustment costs.

The parameters of the process of aggregate shocks, ρ_p and σ_p, are calibrated so that the model approximately replicates the persistence and volatility of (de-trended) log aggregate employment. Using postwar quarterly time series on private payroll employment, and detrending using an HP filter with smoothing parameter 10^5, we compute an autocorrelation coefficient of 0.96 and a standard deviation of 0.025. Values of $\rho_p = 0.95$ and $\sigma_p = 0.015$ are roughly consistent with these moments (see Table 1). We do this because our goal is not to explain the volatility of aggregate employment, but to compare model outcomes within an environment that is economically relevant. One way of doing that is to generate aggregate outcomes that are comparable to what we observe in the data.

Lastly, as noted at the conclusion of Section 2, we have generalized the analysis of Sections 3 and 4 to allow for worker attrition. Accordingly, we have incorporated a constant rate of attrition, δ, into our quantitative analysis. To calibrate δ, we use the simple average of the quarterly quit rate from the Job Openings and Labor Turnover Survey. This is 6%.

As stressed in Proposition 2, approximate neutrality obtains for any given aggregate state, which includes the wage, and thus is not an outcome of equilibrium price adjustment. It is, instead, an aggregation result that relies only on the symmetry in the distributional dynamics. To emphasize this point, we simulate the model for a *fixed* wage. The latter is chosen to induce an average firm size of 20, which is in line with evidence from the Census' Business Dynamics Statistics.[14] Appendix D of the Supplementary Material discusses how to implement the model in general equilibrium and presents impulse responses in this case. The results for the baseline parameterization are virtually identical to what we present here.

Since the wage is fixed, firms do not need to forecast future wages. This means, in turn, that they do not need to forecast future employment distributions. Therefore, the aggregate state Ω is summarized completely by aggregate productivity p, and the optimal policy functions take the simple form $L(n;p)$, $X(n;p)$, and $U(n;p)$. As we noted in Section 2, a positive innovation to aggregate productivity p shifts these functions downward—for a given level of idiosyncratic productivity, a firm is more likely to hire, less likely to fire, and will select a higher level of employment conditional on adjustment. Thus, the evolution of aggregate productivity p induces shifts in the policy function, which, via the law of motion (9), trace out the evolution of the distribution of employment and thereby aggregate employment.

The results of this exercise under the baseline calibration are illustrated in Figs. 2 and 3. We begin in Fig. 2 by analyzing the properties of the steady-state distribution of employment that would be attained in the absence of aggregate shocks. The latter is compared to two reference distributions. The first is the frictionless distribution. The second is the distribution induced by a myopic labor demand policy, in reference to Lemma 2.

Fig. 2 reveals that the steady-state distribution of employment mimics closely its myopic and frictionless counterparts at virtually all employment levels. As foreshadowed by the discussion in Section 4.3 highlighting the important role of heterogeneity and two-sided adjustment, any deviations that do emerge are restricted to very small firm sizes of fewer than two workers. Moreover, these discrepancies are very small in practice. Fig. 2 thus reveals that the neutrality of the

[14] See http://www.census.gov/ces/dataproducts/bds/data_estab.html. We compute the average firm size over the full sample for the years 1977–2011.

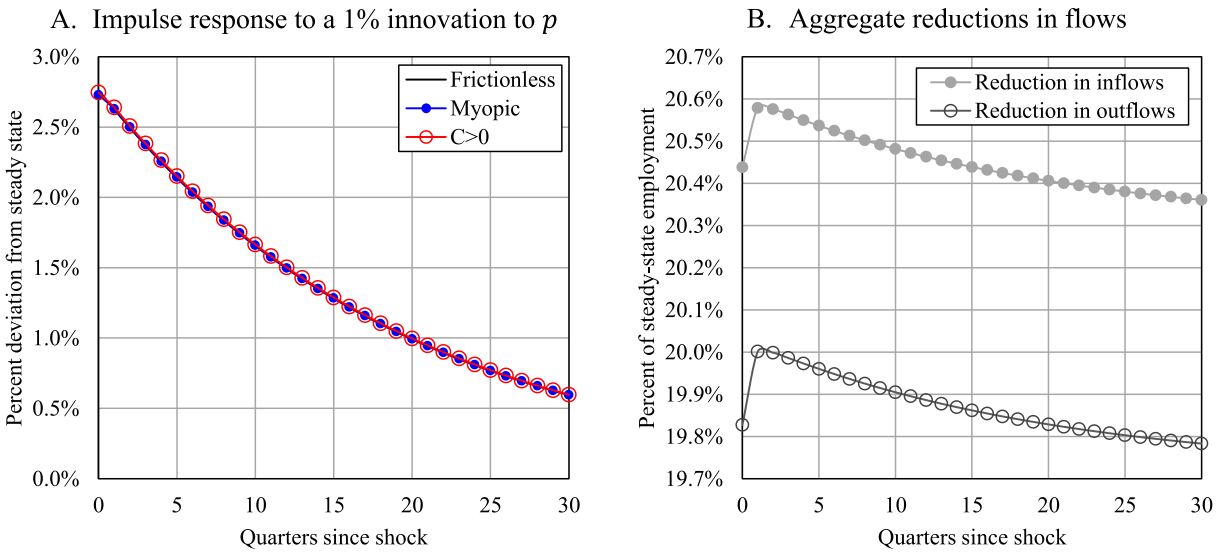

Fig. 3. Dynamic response in baseline parameterization.

dynamics of the firm size distribution implied by Proposition 2 also holds in steady state for conventional parameterizations of an employment adjustment problem.

In Fig. 3 we turn to the dynamic implications of the model. Panel A presents the impulse response of aggregate employment to a one-percent positive innovation to aggregate labor productivity p implied by the baseline parameterization, and contrasts it with its frictionless ($C = 0$) and myopic ($\beta = 0$ and $C > 0$) counterparts. The differences between the impulse responses are so small as to be almost imperceptible. Thus, the prediction of approximate dynamic neutrality in Proposition 2 is not merely a theoretical curiosity; it holds under an empirically-relevant set of parameters.

The source of this approximate neutrality is illustrated in panel B of Fig. 3. This exercise is informed by the emphasis of Proposition 2 on the symmetry of the effects of adjustment frictions on the flows in and out of the mass at each employment level. In particular, rearranging the identity in Eq. (14), multiplying through by n, and integrating yields the following description of the relation between actual and frictionless aggregate employment:

$$N = N^* + \int n[\text{reduction in outflows}(n)]\mathrm{d}n - \int n[\text{reduction in inflows}(n)]\mathrm{d}n. \tag{25}$$

Here $N \equiv \int nh(n)\mathrm{d}n$ is aggregate employment in the baseline (forward-looking) model and $N^* \equiv \int nh^*(n)\mathrm{d}n$ is its mandated counterpart. The aggregate effects of the adjustment cost are thus mediated by the final two terms on the right-hand side of (25). These represent the employment-weighted reductions, relative to the frictionless model, in the flows in and out of each employment level. Panel B of Fig. 3 plots the impulse responses of these two terms, normalized by pre-impulse aggregate employment.[15]

Two results emerge from the exercise in Fig. 3B. First, the aggregate reductions in the inflows and outflows induced by the fixed cost are substantial. At their peak, each amounts to about 20% of steady-state employment. In this sense, the fixed adjustment cost does disrupt significantly the flows to and from each point along the distribution. Second, as predicted by the neutrality result in Proposition 2, the effect of the adjustment cost on the inflows is almost perfectly offset by its effect on the outflows. At no point does the difference exceed 0.6 of one percent. Moreover, the two series move in tandem. This illustrates the symmetry in the distributional dynamics that underlies the approximately frictionless aggregate dynamics in the model.

Interestingly, these quantitative results dovetail with recent literature on dynamic factor demand that has solved numerical models of fixed adjustment costs under specific parametric assumptions. Our finding that aggregate dynamics are approximately invariant with respect to the fixed cost evokes the findings of Cooper et al. (1999) and Cooper and Haltiwanger (2006), whose estimates imply that, in the case of capital adjustment, aggregation smooths away much of the effect of nonconvex adjustment frictions.

[15] Formally, these are calculated by generalizing the i.i.d. case, expressed in (15), to account for persistent shocks. For example, the reduction in outflows is computed as $h_{-1}(n)(\mathcal{G}[U(n)|n] - \mathcal{G}[L(n)|n])$.

5.2. Sensitivity analysis

In this section, we investigate the robustness of the results presented thus far. We consider plausible variations on the baseline parameterization based on six experiments.

5.2.1. Raising C relative to σ_x

The first two experiments investigate the effects of alternative choices of the adjustment cost C and the dispersion of idiosyncratic shocks σ_x. These exercises are motivated by the discussion of Section 4.3, which highlights the crucial role of the magnitude of C relative to σ_x in the neutrality result in Proposition 2.

Panel A of Fig. 4 considers the effects of increasing C so that the adjustment cost is 16% of revenue, on average, across firms. This corresponds to a two-standard error increase above Bloom's (2009) estimate. Likewise, in panel B of Fig. 4, we lower the standard deviation of innovations to idiosyncratic productivity σ_x to 0.2, in line with the lower end of estimates in the literature surveyed in Section 5.1. To hold all else equal, for Panel B we adjust C so that it continues to equal 8% of revenue, on average. As before, we compare these impulse responses to their frictionless counterparts, and illustrate the corresponding reductions in the constituent flows outlined in equation (25).[16]

Both of these experiments lower rates of adjustment: Average quarterly adjustment probabilities are 44% in the parameterization underlying Fig. 4A, and 28% in that underlying Fig. 4B. This greater degree of inaction is in turn reflected in the impulse responses in Fig. 4A and B. The latter in particular reveals a modest hump-shape, with a peak response after just one quarter, and almost frictionless dynamics thereafter. The contrast with Fig. 3 is consistent with our interpretation of Proposition 2, which revealed that symmetry is likely to fail if productive heterogeneity is more limited relative to the adjustment friction. But, the magnitudes of the deviations remain small.

5.2.2. Matching the frequency and size of adjustments

The latter experiments have counterfactual implications for rates of employment adjustment, however. As noted above, the empirical rate of employment adjustment is much higher than that underlying Fig. 4B, at 48.5% in U.S. establishment-level data. For this reason, in our third experiment we explore the effects of calibrating the adjustment cost C and the dispersion of idiosyncratic shocks σ_x to target two salient moments of the cross-establishment distribution of employment growth: the average quarterly frequency of adjusting of 48.5%; and the average absolute quarterly log change in employment among adjusters, which is 0.31.[17] This exercise significantly reduces the adjustment cost to just 0.36% of average quarterly revenue, as well as the degree of idiosyncratic dispersion σ_x, which falls to 0.08.

Panel C of Fig. 4 presents the results of this experiment. Reiterating the important role of the rate of adjustment in the approximations underlying Proposition 2, Fig. 4C reveals that this alternative calibration strategy largely restores the neutrality result noted in the baseline case in Fig. 3: the impulse response is almost indistingushable from the frictionless analogue. The message of this experiment is that Proposition 2 is quantitatively relevant in a calibration that replicates key aspects of the cross section of employment growth.

5.2.3. Varying idiosyncratic persistence, ρ_x

We noted earlier that leading estimates of the persistence of idiosyncratic productivity shocks ρ_x vary widely across studies. A common intuition is that firms should adjust less aggressively to idiosyncratic shocks if productivity is more transitory in order to position employment so it is optimal given expected future reversion to mean in productivity. However, the myopic approximation in Lemma 2 suggests the payoff to this foresight is small. For this reason, Proposition 2 suggests that the lack of empirical consensus over ρ_x is inessential to the presence or otherwise of approximate aggregate neutrality—the result holds independently of ρ_x. Motivated by this, in a fourth experiment we consider the effects of lowering ρ_x to 0.4 (in line with the majority of Cooper et al.'s estimates), and of raising ρ_x to 0.9 (closer to the estimates of Foster et al.). Panel D of Fig. 4 illustrates the results and confirms the predictions of Proposition 2: Changing ρ_x has almost no effect on the impulse response of aggregate employment, which continues to track its frictionless path.

5.2.4. Stochastic adjustment costs

Our baseline model assumes a lump-sum fixed cost, C. A common alternative specification adopted in recent literature is one whereby the adjustment cost is drawn each period from a given distribution. It is straightforward to incorporate such stochastic fixed costs into the above model and to (re-)prove our propositions. Suppose that fixed costs are drawn from a distribution with upper support, $\bar{C}$. If $\bar{C}$ is small (in the sense discussed in Section 4), then the approximation to the adjustment triggers in Lemma 1 can be applied for any $C < \bar{C}$. Moreover, under this assumption, the order-of-magnitude argument behind the optimality of myopia in Lemma 2 also is preserved. As a result, one can adapt the approach of Section 4 to show that, to a first-order approximation, the neutrality result in Proposition 2 remains intact.

[16] To avoid clutter, we omit the impulse responses generated by the myopic model. In each case, these are very similar to the impulse responses in the baseline model.

[17] Thanks to David Ratner, who provided these estimates from BLS Business Employment Dynamics (BED) microdata. The latter record quarterly employment for nearly 75% of U.S. establishments.

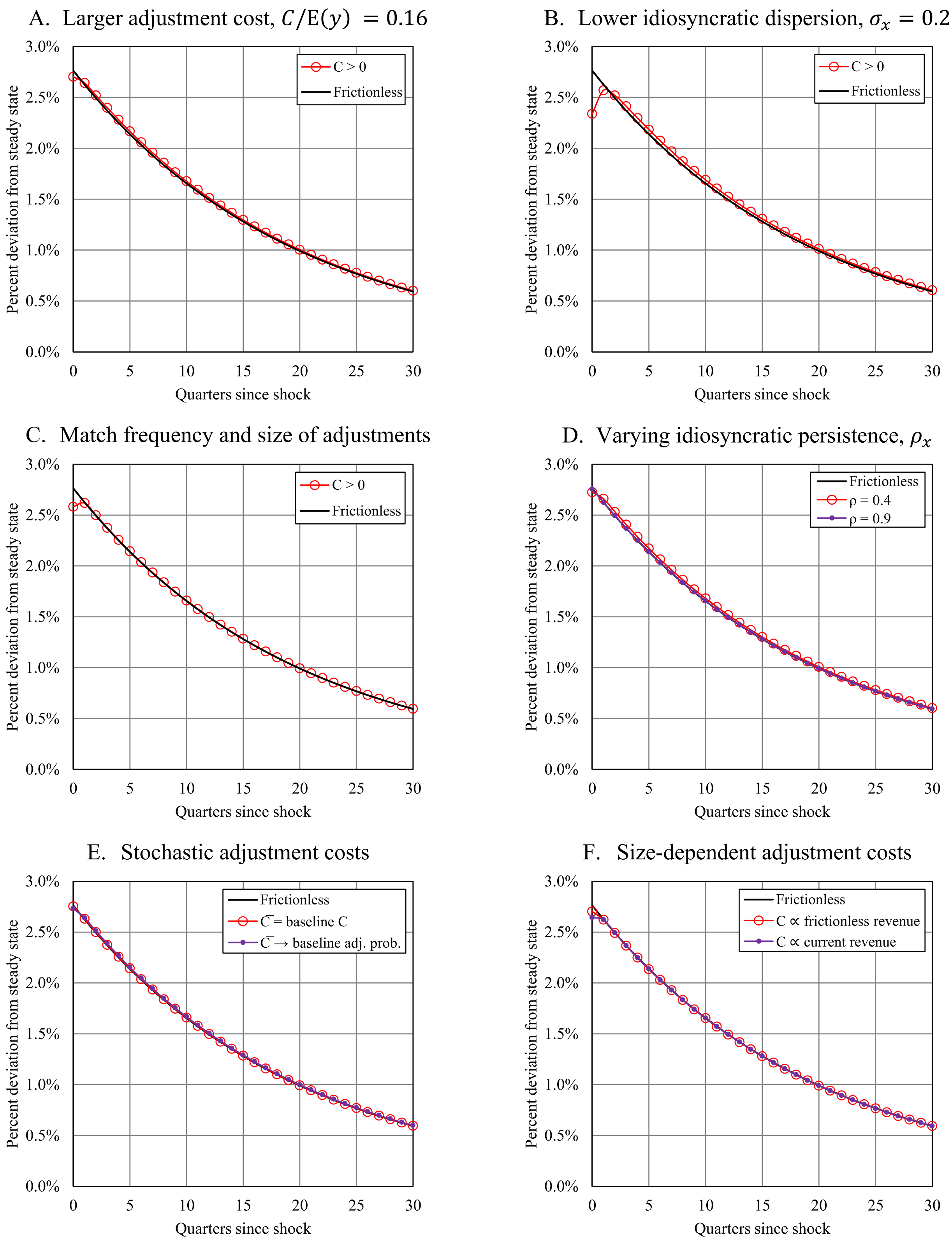

Fig. 4. Sensitivity analysis.

To pursue this argument further, Fig. 4E plots the implied impulse responses for aggregate employment from a version of the baseline model in which firms take i.i.d. draws of fixed costs from a uniform distribution bounded below by 0 and above by $\bar{C}$, as in King and Thomas (2006). All other parameters in the baseline case are retained. We consider two parameterizations of $\bar{C}$. The first sets $\bar{C}$ to the value of the lump-sum fixed cost used in the baseline calibration. The second chooses $\bar{C}$ so that the average probability of adjusting coincides with its value in the baseline calibration. The results of Fig. 4E confirm that the presence of stochastic fixed adjustment costs *per se* has little effect on the baseline results.

5.2.5. Size-dependent adjustment costs

A second alternative specification of adjustment costs used in recent literature has been to scale these costs by some measure of firm size, so that firms do not outgrow the friction.[18] Two common approaches have been implemented. First, Caballero and Engel (1999) and Gertler and Leahy (2008) scale the adjustment cost to be proportional to frictionless revenue, $C = cy^{\star}(x) \equiv cxF(n^{\star}(x))$. In a second specification, the adjustment cost is modeled as a share of *current* revenue, $C = cxF(n)$. This is the specification used in Cooper et al. (2007, 2015), Bloom (2009), and Bachmann (2013). Note that these cases imply a certain asymmetry to the adjustment cost function.

Consider first the simpler case of $C = cy^{\star}(x)$. Since this form of the adjustment cost is independent of the firm's choice of employment, its qualitative implications for the policy rule (Lemmas 1 and 2) and thereby for the approximate neutrality of the implied aggregate dynamics will be expected to mirror those derived above for the case of a lump-sum cost. Fig. 4F confirms this expectation. It presents the implied impulse response in the case where c is set to replicate the average adjustment rate in the baseline parameterization illustrated in Fig. 3. It is almost indistinguishable from the frictionless response.

This extended result in turn aids interpretation of the more complicated case in which $C = cxF(n)$. The latter is increasing in the choice of employment. It follows that the adjustment cost distorts the optimal level of employment *conditional on* adjusting, $n = X^{-1}(x; c)$, because it acts like a tax on increases in n. Thus, the key difference in this model is that the distribution of mandated employment implied by this distorted reset policy will diverge from its frictionless analogue. All other features resemble the case above where $C = cy^{\star}(x)$. Therefore, it is natural to expect neutrality to obtain with respect to the path of mandated, but not frictionless, aggregate employment. However, the deviations from the frictionless path are quite small, as shown in Figure 4F. Relative to the simpler size-dependent case, a slight deviation emerges on impact, but this is subsequently eliminated.

5.3. Relation to the literature

What emerges from the foregoing quantitative analysis is that the presence of a fixed adjustment cost has, at most, only a modest effect on aggregate dynamics under reasonable parameterizations, even in the absence of adjustment of market prices. As in Proposition 2, the source of these limited effects can be traced to the symmetric role of the adjustment cost in reducing the flows in and out of each position in the cross section. And, where (small) deviations in aggregate dynamics do arise, it is in parameterizations that imply rates of adjustment significantly lower than those seen in microdata on employment.

These observations share parallels in prior literature based on numerical work. For instance, King and Thomas (2006) document deviations of aggregate dynamics with respect to the frictionless case when market prices are fixed, as they are in the simulations reported above. Gourio and Kashyap (2007) report similar quantitative findings in their analysis of a related investment problem. For simplicity, however, the sole source of heterogeneity in both of these analyses is (modest) variation in the form of a stochastic fixed cost of adjustment; both studies abstract entirely from productive heterogeneity, implicitly imposing that $\sigma_x = 0$. The foregoing analysis thus suggests that the non-neutralities found in these earlier studies are a consequence of the assumed absence of idiosyncratic heterogeneity.[19]

Important precedents in prior literature do allow for productive heterogeneity, however. Khan and Thomas (2008) provide a calibration of a related investment model that successfully confronts several features of the data on plant-level investment. The implied dispersion in productivity σ_x is such that adjustment rates are significantly lower than in our baseline case above.[20] As foreshadowed by the interpretation of Proposition 2, and the quantitative analysis in Fig. 4B, Khan and Thomas find that deviations emerge between frictionless dynamics and the behavior of aggregate capital in the presence of the adjustment costs, if market prices are fixed. This suggests that calibrations similar to that summarized in Fig. 4B may be relevant to the case of capital adjustment.

Our own analysis suggests that higher adjustment rates are more relevant for the case of labor demand, and that these in turn imply aggregate dynamics almost indistinguishable from their frictionless counterpart. Interestingly, our results

[18] However, the probability of adjusting employment in BLS Business Employment Dynamics micro data does increase in establishment size. One interpretation is that it is consistent with a lump-sum friction. By contrast, formalizations of size-dependent costs typically imply that firms are never large relative to the adjustment cost, and thus fail to replicate this fact.

[19] Bachmann (2013) also finds that a fixed adjustment cost model induces sluggish dynamics in aggregate employment. While his model allows for idiosyncratic risk comparable to that used in this paper, his calibration still implies a comparatively low adjustment rate.

[20] For example, Khan and Thomas's calibration implies that 75% of plants would not adjust their capital stock in a given year, but for the fact that their model exempts very small adjustments from the adjustment cost.

also suggest that estimates of the frequency of price adjustment imply very limited non-neutrality. The recent survey of Klenow and Malin (2011) suggests that, after omitting many sales-related price changes, the mean (median) duration of prices is about 7 (5.9) months. If price changes coinciding with product substitutions are excluded, the mean (median) duration rises to 10 (8.3) months.[21] This range from 7 to 10 months is encompassed by Fig. 4B and C; the latter implies a mean duration of employment of a little more than 6 months, whereas the former yields a duration of almost 10.5 months. Thus, recalling the isomorphism between our model and price-setting problems, we infer that leading estimates of the frequency of price changes suggest dynamics of the aggregate price level that lie between the impulses responses in Fig. 4B and C. This represents a rather small departure from neutrality.

5.4. Generating non-neutralities: an analytical illustration

We close this section by highlighting how the analytical framework provided in this paper can help elucidate the sources of *non*-neutralities. An influential strand of recent research has argued that the form of idiosyncratic shocks plays a crucial role in shaping the aggregate effects of lumpy microeconomic adjustment. In particular, Gertler and Leahy (2008) and Midrigan (2011) have studied environments in which idiosyncratic shocks evolve according to a compound Poisson process whereby individual firms receive a shock with probability $1-\lambda$ each period. Interestingly, they find that this departure gives rise to persistent aggregate dynamics, in contrast to the results of previous sections of this paper.[22]

In what follows, we show that the analysis and intuition of Sections 3 and 4 provide a novel perspective on the origins of this result. In particular, we are able to trace this result analytically to a clear violation of symmetry in the distributional dynamics.

For clarity, consider the case in which idiosyncratic shocks are conditionally i.i.d. That is, with probability $1-\lambda$ each period firms receive an independent draw x' from a distribution function $G(x')$, while with probability λ no idiosyncratic shock arrives and $x' = x$.

As in Section 4, our aim is to approximate the reductions in the flows in and out of the mass $h(n)$ relative to a frictionless world in which all firms adjust every period.[23] Note that these flows essentially are unchanged for the set of firms that receive an idiosyncratic shock. What is different is that there exists a mass of firms that receive no idiosyncratic shock, but may adjust to aggregate shocks.

In their model of menu costs, Gertler and Leahy (2008) show that almost none of the latter firms in fact adjusts in the presence of plausibly small aggregate disturbances. The same is true of our model. To understand why, it is helpful first to imagine the model in the absence of aggregate shocks. In that case, a firm that receives no idiosyncratic shock has no reason to adjust: If their current productivity $x = x_{-1}$ lies outside of the inaction region $[L(n_{-1}), U(n_{-1})]$, then it must also have done in the past, and the firm already will have adjusted. All that changes in the presence of aggregate shocks is that the current period's adjustment triggers may differ from the previous period's, inducing some firms on the margin to adjust. When aggregate shocks are small relative to the inaction region, the latter measure of firms will be small.[24]

It follows that the reduction in the outflow from n relative to the frictionless case is approximated by $h_{-1}(n)\{(1-\lambda)(G[U(n)] - G[L(n)]) + \lambda\}$: Of the $1-\lambda$ firms that receive an idiosyncratic shock, a fraction $G[U(n)] - G[L(n)]$ will not adjust away from n; and a share λ receives no idiosyncratic shock and also does not adjust. Similarly, the reduction in the inflow into n is approximated by $h^*(n)\left\{(1-\lambda)\left(H_{-1}\left[L^{-1}X(n)\right] - H_{-1}\left[U^{-1}X(n)\right]\right) + \lambda\right\}$.

Comparison of the latter with the analysis of the continuous-shock case in Section 4 reveals the mechanism at the heart of the persistence induced by the Poisson model. As in Section 4, the reductions in the flows associated with firms that receive idiosyncratic shocks approximately cancel in the presence of a small fixed adjustment cost. What remain are the terms associated with firms that have not received an innovation to x. Crucially, these flows do not cancel. As a result, the implied approximate aggregate dynamics are[25]

$$\Delta h(n) \approx -(1-\lambda)[h_{-1}(n) - h^*(n)]. \tag{26}$$

What emerges, then, is that aggregate dynamics in the presence of Poisson shocks are approximated by a *pure* partial-adjustment process, with convergence rate equal to the probability of receiving an idiosyncratic shock, $1-\lambda$. Equivalently,

---

[21] These estimates are taken from Klenow and Malin's Table 7. The lower end of this range (7 months) is found by comparing "like" prices. This approach retains observations on sales-related price changes only if the current sale price differs from the most recent sale price; sale and non-sale prices are never compared. Mean (median) duration rises to 8 (6.9) months if all sales-related price changes are dropped.

[22] Our analysis abstracts from other dimensions of Midrigan (2011) model, in particular the presence of multi-product firms and associated economies of scope in price adjustment. Midrigan shows that the latter also contribute to non-neutralities.

[23] A subtle but important point is that, even though firms receive idiosyncratic shocks with probability $1-\lambda < 1$, they still adjust every period in a frictionless world due to the presence of aggregate shocks.

[24] By the same token, among firms with $x = x_{-1}$, a discrete mass will have adjusted in the past and will inherit an employment level of $n_{-1} = X^{-1}(x_{-1})$. It follows that aggregate shocks that shift the reset function $X(\cdot)$ enough to induce even these firms to adjust in the current period will induce a discretely-large fraction of firms to adjust. Thus, large aggregate shocks will be more likely to induce neutrality in the presence of Poisson shocks. Karadi and Reiff (2012) investigate this possibility in more detail.

[25] Recall that the frictionless law of motion is $\Delta h(n) = -[h_{-1}(n) - h^*(n)]$. We have shown that the outflows from n are depressed relative to the frictionless case by $\lambda h_{-1}(n)$ and the inflows to n are depressed by $\lambda h^*(n)$. Thus, we can amend the frictionless law of motion to obtain $\Delta h(n) \approx -[h_{-1}(n) - h^*(n)] + \lambda h_{-1}(n) - \lambda h^*(n)$, which yields the expression in the main text.

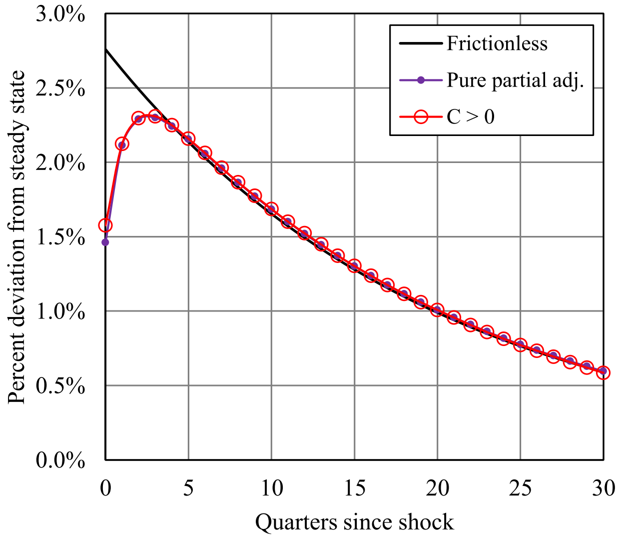

Fig. 5. Compound Poisson idiosyncratic shocks.

the model will behave like a partial adjustment model in which the exogenous probability of adjusting is set to $1 - \lambda$. Since the latter is independent of the level of employment n (in contrast to the continuous-shock case studied above), it follows that aggregate employment will inherit precisely the same partial-adjustment dynamics. Hence, we expect persistent, hump-shaped impulse responses.

To illustrate this point, we calibrate the model with Poisson idiosyncratic shocks and compute the impulse response of aggregate employment to an aggregate productivity innovation. To maximize similarity with the benchmark model, we leave virtually all of the structural parameters unchanged, and modify the adjustment cost to guarantee that it remains equal to 8% of revenue on average. Since any $\lambda > 0$ necessarily lowers the probability of adjusting *ceteris paribus*, however, this calibration will not match the baseline inaction rates. Instead, we compare the Poisson case with a calibration of the benchmark model that implies a comparably small adjustment probability. We find that $\lambda = 0.45$ induces a probability of adjusting in the Poisson model that is similar to that in the low-σ_x parameterization of the benchmark model depicted in Fig. 4B.

Fig. 5 illustrates the results. Consistent with the results of Gertler and Leahy (2008) and Midrigan (2011), one can clearly discern much more persistent aggregate dynamics in this case, with employment converging to its frictionless counterpart after five quarters. Moreover, the persistence cannot be attributed to a lower average adjustment rate—the low-σ_x case in Fig. 4B induces a similar adjustment rate but exhibits much less propagation.

Rather, the persistence is closely linked to the above intuition for the approximate partial-adjustment nature of the model's dynamics in the presence of Poisson shocks. To emphasize this point, Fig. 5 also plots the path of aggregate employment directly from the approximate pure partial-adjustment result in (26) as a point of comparison with the model-generated path. Remarkably, the two paths are almost indistinguishable, suggesting that the approximate analysis above indeed provides a very good guide to the behavior of the model.

The source of this result can be traced to a violation of the symmetry noted in Section 4. There we highlighted the dual, symmetric roles of the distributions of inherited and desired employment, $h_{-1}(n)$ and $h^{*}(n)$, in delivering aggregate neutrality in the presence of continuous shocks. For instance, while it seems clear that $h_{-1}(n)$ is indicative of the mass of firms that is deterred from adjusting *away* from n, a more subtle point is that it also contributes to the size of the reduction in the probability of adjusting *to* n. The reason is that firms whose initial employment is near n (mass in the neighborhood of $h_{-1}(n)$) do not find it optimal to adjust to that position. Hence, what underlies this latter, symmetric effect is the fact is that a firm's propensity to adjust (to n) depends on its initial size. The model with Poisson shocks breaks this symmetry because the arrival of new idiosyncratic shocks is independent of the firm's state. As a result, a fraction of firms does not adjust *regardless* of their initial employment, a feature reminiscent of the partial adjustment model.

6. Summary and discussion

Our analysis of a canonical model of fixed employment adjustment costs has established a stark neutrality result. In general, the dynamics of aggregate employment in the presence of an adjustment friction can be inferred simply and intuitively by characterizing the evolution of the distribution of employment across firms. We show that aggregate employment dynamics approximately coincide with their frictionless counterpart, even in the absence of equilibrium adjustment of market prices. This result arises from a form of symmetry in the dynamics of the firm-size distribution that emerges as the adjustment cost becomes small. In that neighborhood, we show that the reduction in the flow of firms that adjusts to a given employment level is approximately offset by the reduction in the flow of firms that adjusts away from that level, leaving the path of the firm-size distribution almost unimpaired.

Thus, our analysis provides an analytical foundation to recent quantitative research on the macroeconomic effects of discrete adjustment costs in a general framework. It provides a precise formal justification for the approximate neutrality noted in numerical simulations by Golosov and Lucas (2007) in the context of a related menu cost model. Similarly, our own quantitative analysis of a model of employment adjustment calibrated to leading estimates of adjustment costs imply aggregate dynamics that are close to frictionless outcomes, also in line with our approximate neutrality result.

Our analysis also offers a novel perspective on the circumstances in which aggregate dynamics can be expected to deviate from their frictionless counterparts. A unifying theme in our findings is the important role of *symmetry* in unwinding the aggregate effects of lumpy adjustment. It follows that deviations from frictionless dynamics can be traced to violations of this symmetry. We show that an important example of the latter is recent research that has invoked compound Poisson processes of idiosyncratic shocks in which only a fraction $1-\lambda$ of firms receives a shock each period (Gertler and Leahy, 2008; Midrigan, 2011). Our approximations provide a novel perspective on this result: we demonstrate that implied aggregate dynamics in this case are approximately isomorphic to partial adjustment with exogenous adjustment parameter $1-\lambda$.

These results highlight a number of interesting avenues for future research. First, since the magnitude of adjustment costs and idiosyncratic risk play a role in the model's aggregate dynamics, it remains important for empirical work to focus on obtaining robust estimates of these two critical parameters. Second, we join the influential recent work of Gertler and Leahy and Midrigan in emphasizing the role of the *form* of idiosyncratic productivity shocks. Given its theoretical importance, future empirical work that estimates the distribution of idiosyncratic shocks will be of particular value.

To the extent that estimates of these parameters line up with the approximate aggregate neutrality we identify, it is worthwhile to consider other adjustment frictions that simultaneously can account for lumpy microeconomic adjustment and persistent aggregate dynamics. For instance, both fixed and kinked (proportional) adjustment costs induce inaction at the microeconomic level, but may have very different implications for aggregate employment dynamics. In addition, there may be additional frictions, or technological constraints, to which the firm is subject that interact with adjustment costs. For instance, Bachmann et al. (2013) consider a model in which there are "core components " to the capital stock whose depreciation must be replaced in order for the plant to operate. They argue that this feature can amplify the effects of a fixed cost of capital adjustment on the aggregate dynamics of investment.

Our framework would suggest that, to the extent these other frictions alter the dynamics, they must disrupt the symmetry of the adjustment policy. And indeed, using plant-level data on employment and investment, the analysis of Caballero et al. (1995, 1997) does suggest that asymmetries are important empirically. The question of what lies behind this asymmetry—and what it implies for the aggregate dynamics—is thus an important topic for future research.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jmoneco.2018.07.008.

References

Abraham, A., Kirk, W. 2006. "The Dynamics of Plant-Level Productivity in U.S. Manufacturing." Center for Economic Studies WP 06-20.
Akerlof, G.A., Yellen, J.L., 1985. A near-rational model of the business cycle, with wage and price inertia. Q. J. Econ. 823–838. 100(Supplement)
Alvarez, F., Lippi, F., 2014. Price setting with menu cost for multiproduct firms. Econometrica 82 (1), 89–135.
Alvarez, F., LeBehin, H., Lippi, F., 2016. The real effects of monetary shocks in sticky price models: a sufficient statistic approach. Am. Econ. Rev. 106 (10), 1–37.
Bachmann, R., 2013. Understanding jobless recoveries. Mimeo, University of Michigan.
Bachmann, R., Caballero, R.J., Engel, E.M.R.A., 2013. Aggregate implications of lumpy investment: new evidence and a DSGE model. Am. Econ. J. 5 (4), 29–67.
Bertola, G., Caballero, R.J., 1990. Kinked adjustment costs and aggregate dynamics. NBER Macroeconomics Annual 1990. MIT Press, Cambridge, MA.
Bertola, G., Caballero, R.J., 1994. Irreversibility and aggregate investment. Rev. Econ. Stud. 61 (2), 223–246.
Barro, R.J., 1972. A Theory of Monopolistic Price Adjustment. Rev. Econ. Stud. 39 (1), 17–26.
Bertola, G., Guiso, L., Pistaferri, L., 2005. Uncertainty and consumer durables adjustment. Rev. Econ. Stud. 72 (4), 973–1007.
Bils, M., Klenow, P.J., 2004. Some evidence on the importance of sticky prices. J. Polit. Econ. 112 (5), 947–985.
Bloom, N., 2009. The impact of uncertainty shocks. Econometrica 77 (3), 623–685.
Caballero, R.J., Engel, E.M.R.A., 1999. Explaining investment dynamics in u.s. manufacturing: a generalized (s,s) approach. Econometrica 67 (4), 783–826.
Caballero, R.J., Engel, E.M.R.A., Haltiwanger, J.C., 1995. Plant-level adjustment and aggregate investment dynamics. Brookings Pap. Econ. Act. 1995 (2), 1–39.
Caballero, R.J., Engel, E.M.R.A., Haltiwanger, J.C., 1997. Aggregate employment dynamics: building from microeconomic evidence. Am. Econ. Rev. 87 (1), 115–137.
Caplin, A.S., Spulber, D.F., 1987. Menu costs and the neutrality of money. Q. J. Econ. 102 (4), 703–725.
Clausen, A., Strub, C., 2016. A general and intuitive envelope theorem. Mimeo, University of Edinburgh.
Cooper, R., Haltiwanger, J., Willis, J.L., 2007. Search frictions: matching aggregate and establishment observations. J. Monet. Econ. 54 (1), 56–78.
Cooper, R., Haltiwanger, J., Willis, J.L., 2015. Dynamics of labor demand: evidence from plant-level observations and aggregate implications. Res. Econ. 69 (1), 37–50.
Cooper, R.W., Haltiwanger, J.C., 2006. On the nature of capital adjustment costs. Rev. Econ. Stud. 73 (3), 611–633.
Cooper, R.W., Haltiwanger, J.C., Power, L., 1999. Machine replacement and the business cycle: lumps and bumps. Am. Econ. Rev. 89 (4), 921–946.
Davis, S.J., Haltiwanger, J., 1992. Gross job creation, gross job destruction, and employment reallocation. Q. J. Econ. 107 (3), 819–864.
Dixit, A.K., 1991. Analytical approximations in models of hysteresis. Rev. Econ. Stud. 58 (1), 141–151.

Dixit, A.K., 1993. The art of smooth pasting. In: Lesourne, J., Sonnenschein, H. (Eds.), Vol. 55 of Series Fundamentals of Pure and Applied Economics. Harwood Academic Publishers, Reading, UK.
Dixit, A.K., Pindyck, R.S., 1994. Investment Under Uncertainty. Princeton University Press, Princeton (NJ).
Doms, M.E., Dunne, T., 1998. Capital adjustment patterns in manufacturing plants. Rev. Econ. Dyn. 1 (2), 409–429.
Foster, L., Haltiwanger, J., Syverson, C., 2008. Reallocation, firm turnover, and efficiency: selection on productivity or profitability? Am. Econ. Rev. 98 (1), 394–425.
Gertler, M., Leahy, J., 2008. A phillips curve with an *Ss* foundation. J. Polit. Econ. 116 (3), 533–572.
Golosov, M., Lucas, R.E., 2007. Menu costs and phillips curves. J. Polit. Econ. 115 (2), 171–199.
Gourio, F., Kashyap, A.K., 2007. Investment spikes: new facts and a general equilibrium exploration. J. Monet. Econ. 54(Supplement) 1–22.
Hamermesh, D., 1989. Labor demand and the structure of adjustment costs. Am. Econ. Rev. 79 (4), 674–689.
Harrison, J.M., Sellke, T.M., Taylor, A.J., 1983. Impulse control of Brownian motion. Math. Oper. Res. 8 (3), 454–466.
House, C.L., 2014. Fixed costs and long-lived investments. J. Monet. Econ. 68, 86–100.
Karadi, P., Reiff, A., 2012. Large shocks in menu cost models. Working Paper No. 1453. European Central Bank.
Khan, A., Thomas, J., 2008. Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics. Econometrica 76 (2), 395–436.
King, R., Thomas, J., 2006. Partial adjustment without apology. Int. Econ. Rev. 47 (3), 779–809.
Klenow, P.J., Malin, B.A., 2011. Microeconomic evidence on price-setting. In: Friedman, B.M., Woodford, M. (Eds.), Handbook of Monetary Economics, 3, pp. 231–284.
Mankiw, N.G., 1985. Small menu costs and large business cycles: a macroeconomic model of monopoly. Q. J. Econ. 100 (2), 529–537.
Midrigan, V., 2011. Menu costs, multi-product firms, and aggregate fluctuations. Econometrica 79 (4), 1139–1180.
Roys, N., 2014. Optimal investment policy with fixed adjustment costs and complete irreversibility. Econ. Lett. 124 (3), 416–419.
Scarf, H., 1959. The optimality of (s, s) policies in the dynamic inventory problem. In: Arrow, K.J., Karlin, S., Suppes, P. (Eds.), Mathematical Methods in the Social Sciences. Stanford University Press, Stanford.
Veracierto, M.L., 2002. Plant-level irreversible investment and equilibrium business cycles. Am. Econ. Rev. 92 (1), 181–197.